Dynamic Pitching Moment Measurement of a Wing Executing Pitch and Plunge Motion

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This work is an attempt to extract the dynamic pitching moment behavior of a finite wing through stall including rate effects, purely from the kinematics of the wing. A finite wing model, free to pitch about its quarter-chord, is driven through large-amplitude plunge oscillations in the open jet of a low speed wind tunnel. The history of the pitch and plunge motions is used to derive lift and pitching moment behavior, through a range of rates. The error in the quasi-steady, linear phases of the experiment is estimated by comparison with potential flow calculations and dynamic simulation. Quasi-steady lift is seen to be captured well, and opens the way to a wide range of angle of attack range. Variation of the lift curve slope with Reynolds number shows surprising effects in the transitional range. Pitching moment results seem promising, but some anomalies remain to be explained. These will be refined for the final paper. Preliminary results in the dynamic regime are shown.

I. Nomenclature

\[ A = \text{amplitude of oscillation [m]} \]
\[ AR = \text{aspect ratio} \]
\[ c = \text{chord [m]} \]
\[ c_l = \text{coefficient of lift} \]
\[ c_m = \text{coefficient of moment} \]
\[ I = \text{Moment of Inertia [Kg m}^2\text{]} \]
\[ l_a = \text{length of arm [m]} \]
\[ L = \text{lift [N]} \]
\[ m = \text{mass of wing [Kg]} \]
\[ q_\infty = \text{dynamic Pressure [Pa]} \]
\[ S = \text{planform Area [m}^2\text{]} \]
\[ U_\infty = \text{wind tunnel speed [m/s]} \]
\[ \alpha_p = \text{angular acceleration of the wing [rad/sec}^2\text{]} \]
\[ \alpha = \text{angle of attack [degrees]} \]
\[ \dot{\alpha} = \text{angular rotational speed [deg/s]} \]
\[ \ddot{\alpha} = \text{angular acceleration [deg/sec}^2\text{]} \]
\[ \beta = \text{angle of arm [degrees]} \]
\[ \omega = \text{frequency of oscillation [Hz]} \]
\[ \theta = \text{angle of wing [degrees]} \]

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\[ \dot{\theta} = \text{wing rotational speed [deg/s]} \]
\[ \phi = \text{phase angle [degrees]} \]
\[ t = \text{time [s]} \]
\[ f = \text{frequency [Hz]} \]
\[ a_i, b_i = \text{amplitude} \]
\[ \frac{\partial c_l}{\partial \alpha} = \text{lift curve slope} \]
\[ \dot{y} = \text{wing translational speed [m/s]} \]

II. Introduction

A. Background and Problem Statement

Dynamic stall of wings and rotor blades has been a subject of intense interest for several decades. Most of the literature on explaining and predicting dynamic stall arises from interest in rotorcraft forward flight at high advance ratio, post-stall maneuvering of combat aircraft, and operation of jet engine compressors at high stage pressure ratio. Research in this area has progressed from predictions and experiments on oscillating “airfoil sections” in wind and water tunnels,\textsuperscript{1,2,3,4} to rotating blades with inflow, and swept wings undergoing large excursions in angle of attack and roll. Extensive studies of dynamic stall on oscillating wings have been accomplished.\textsuperscript{5,6,7} Numerical simulations have proceeded from inviscid formulation to full Navier-Stokes simulations.\textsuperscript{8,9}

The pitching moment behavior through stall is perhaps the aspect of greatest practical interest in these studies, because the associated loads are severe. Studies\textsuperscript{10,11} correlating flight data with predictions show that the event of primary interest to load prediction is the large excursion in blade pitching moment that accompanies the separation and convection of the dynamic stall vortex.

Capturing the pitching moment from calculations or from direct load measurements is quite difficult and expensive. Traditional approaches use complex motion rigs involvingcams, levels and robotic arms. There is a fundamental issue in selecting load-measuring instrumentation for such experiments, since most such instruments involve deflection of some elastic member. A careful trade-off is required between sensitivity and dynamic range, to enable successful measurements without compromising safety of the experiment and the instrumentation. These issues drive the costs of such experiments to high levels, so that not enough experiments can be performed.

An alternative method is investigated here. It is to extract the loads from the observed free motion, measured mass and kinematics of a model, without any direct force or moment sensing. The reasoning is that the measuring system in this case is a simple shaft encoder, whose angular sensitivity and frequency response are excellent over any desired range of motion. With the data acquisition rate being arbitrarily high, and the motion repeatable over many cycles, we reason that continuous, differentiable data can be obtained with excellent accuracy, on the motion, and therefore on the acceleration history. Knowing all positions, velocities and accelerations and all masses and moments of inertia, the loads can be extracted through kinematics and dynamics calculations. If successful, the technique then opens the way to measuring the nonlinear aerodynamics encountered over wide ranges of angle of attack, and unsteady aerodynamics effects associated with higher rates of motion. As we see in the paper, it also reveals behavior at low Reynolds numbers, that are traditionally difficult to measure in wind tunnels.

B. Strategy

A double pendulum wing is set up, having a pivoted arm with a separately pivoted wing at the end of the arm as shown in Figures 1 and 2. This setup allows for pitch and plunge oscillation modes of a finite wing model. The pivoted arm when set into oscillatory motion results in coupled oscillations of the pivoted arm and the wing. Experiments and simulations are used to extract the behavior of pitching-and-plunging wing.

The strategy detailed in the paper is as follows. We swing an arm through a periodic motion in a wind tunnel freestream, using a driving motor. At the end of the arm is pivoted a finite wing model. The wing can be left to float in pitch about the pivot while the arm is driven. As an alternative means of operation, the model also has a servo motor at the pivot, which when engaged can be used to specify certain types of angle of attack history. This feature can be used to generate torque using the lift on the wing, to drive the
Figure 1. Model of Setup

Figure 2. Picture of Setup
arm which can be left free to swing in that case. In all cases, the measurements taken are the streams of
digital encoder data coming from the shaft at the arm origin, and that at the wing pivot.

As seen in the following, the motions are not exactly repeated from cycle to cycle, since the torque
opposing the motor varies with aerodynamics, and the wing aerodynamics varies widely, especially as we
go to low Reynolds numbers or high rates. This after all, is the problem in predicting the precise timing
of pitching moment stall, that plagues helicopter designers. Our solution is to transform the data to the
frequency regime, and construct auto-power spectra of the motion histories. These lead to stable averages.
The spectral coefficients are then extracted to reconstruct a finite Fourier series representing the motion.
This function is continuous, differentiable and accurate, so that it can be differentiated twice to obtain
acceleration, without significant error.

In short order, the experiment produces a huge amount of data spanning the angle of attack and rate
regimes, with significant Reynolds number effects. Validation poses a problem. We are attacking that
problem systematically with this first paper, where we lay out the kinematics and shape of the setup, and
then check the results in the linear regime where there are data and analysis to use for validation. The low
Reynolds number still poses interesting challenges.

III. Approach

The experiment works as follows: the drive shaft rotates by a belt and pulley system driven by a mounted
motor, which then rotates the airfoil via a connecting arm and creates an oscillatory motion for the airfoil.
The angle of attack is also variable due to another belt and pulley system on the arm itself that is being
driven by a servomotor also attached on the arm. This setup will be mounted in the small wind tunnel
from the floor with the main motor hidden underneath the tunnel resting on a platform. The arms, arms
and rotating mechanism as well as the mechanism to change the angle of attack will actually be in the free
stream. Both a CAD model and a picture are featured as Figures 1 and 2.

The pitch-plunge motion of a wing due to sinusoidal driving of an arm about whose end it is pivoted,
the arm angle and wing angle related through

\[ \alpha = \beta + \theta \] (2)

The motion of the arm and of the wing are captured by streams of digital signals describing the instan-
taneous position of shaft encoders. The position as a continuous function of time may be approximated by
a finite Fourier series, in the form:

\[ x(t) = a_0 + \sum a_i \sin(2\pi f_i t) + b_i \cos(2\pi f_i t) \] (3)

The number of terms necessary to accurately recreate the complex encoder oscillations may be as high as
20 to 30. The amplitude \( a_i, b_i \) at each frequency \( f_i \) are found by taking finite discrete Fourier transforms of
the raw encoder positional data is passed through a Fast Fourier Transform (FFT) performed in MATLAB
software. The real and imaginary components at each frequency \( f_i \) are given as \( b_i \) and \( a_i \) respectively. They

\[ A_i = \sqrt{a_i^2 + b_i^2} \]

\[ \phi_i = \tan^{-1} \left( \frac{a_i}{b_i} \right) \] (5)

An example for a complex oscillatory motion is illustrated in Figure 3. In this test case, the arm was
allowed to swing free, while the wing was driven by a servo with 10 ms time delay, at a wind tunnel freestream
speed of 3.05 m/s (10 ft/s).
When the wing attitude is driven by the servo, its angle of attack changes, thereby generating lift. The component of the force acting perpendicular to the arm, generates a torque which accelerates the swinging of the arm. This in turn drives a plunge motion of the wing, which changes its angle of attack, and causes a pitching moment due to the inertia of the wing as well as aerodynamic pitching moment. Thus the response is more complex than the driving motion.

While the wing oscillates with a fairly regular sinusoidal motion (since it is driven by the servo), the arm motion seems to be composed of multiple sine functions. The power spectra in Figures 4 and 5, averaged over many cycles of the motion, confirm this observation. Figure 4 shows only one dominant frequency peak at 0.2848 Hz for the wing, while Figure 5 shows two dominant frequency peaks at 0.1424 Hz and 0.2848 Hz.

The Fourier coefficients from the averaged spectra are used to reconstruct the motion histories as continuous analytic functions, so that they can be differentiated. The first and second time derivatives are obtained analytically to find $\dot{\theta}$, $\ddot{\theta}$, $\dot{\beta}$, and $\ddot{\beta}$. For the arm angle $\theta$, the derivatives are as follows:

$$\dot{\theta}(t) = A_1 \omega_1 \cos(\omega_1 t - \phi_1) + A_2 \omega_2 \cos(\omega_2 t - \phi_2) + A_3 \omega_3 \cos(\omega_3 t - \phi_3) + ...$$  \hspace{1cm} (6)$$

$$\ddot{\theta}(t) = -A_1 \omega_1^2 \sin(\omega_1 t - \phi_1) - A_2 \omega_2^2 \sin(\omega_2 t - \phi_2) - A_3 \omega_3^2 \sin(\omega_3 t - \phi_3) - ...$$  \hspace{1cm} (7)$$

Likewise, the same procedure is applied to the wing angle. Results from the reconstructed motion are shown in the following plots for the case for the servo driving the wing with 10 ms time delay, an undriven arm with unconstrained sweep angle, and at 3.05 m/s (10 ft/s) wind tunnel speed shown in Figure 6.

The number of terms used for the wing is 5, while for the more complex motion of the arm, 10 terms are used to reconstruct the position over time. For better comparison, Figure 7 shows the reconstructed arm angle juxtaposed against the experimental data.

Figure 8 shows another complex case with the arm driven at 7 amps with a maximum sweep of 30° at 10 ft/s wind tunnel speed. Here, 20 terms are used for both the wing and arm angles.

Once the position in time is expressed as a continuous function, angular velocity and acceleration may be obtained, also at any instantaneous point in time. For the arm angle in Figure 3, these derivatives are
Figure 4. Auto Power Spectrum of the Wing Motion

Figure 5. Auto Power Spectrum of the Arm Motion
Figure 6. Motion history sample reconstructed from the averaged spectra of the wing and arm motions.

Figure 7. Comparison of reconstructed motion history to a single sample of the motion from raw encoder data.
Figure 8. 20-term representation of a case with arm driven at 7 Amps with maximum sweep of 30\(^\circ\) at 3.05m/s freestream speed.

shown in Figures 9 and 10.

A. Lift Coefficients

The equation for lift coefficient is given as the product of the effective angle of attack and a non-constant 2D lift-curve slope. The effective angle of attack is composed of the instantaneous component, a rotational component due to pitching, and a translational component due to plunging.

\[
cl = \frac{dc_l}{d\alpha} [\alpha - \frac{c\dot{\alpha}}{2U_\infty} - \tan^{-1}(\frac{\dot{y}}{U_\infty})]
\]

\[
\alpha = \theta + \beta
\]

The instantaneous angle of attack \(\alpha\) is the sum of the arm and wing angles \(\theta\) and \(\beta\) gathered directly from the positional encoders. \(\theta\) is the angle of the arm with respect to the axis of symmetry parallel to the freestream wind tunnel flow. \(\beta\) is the angle of the wing with respect to the arm. \(\dot{y}\) in the translational term may be obtained from the encoders through Equation 10.

\[
\dot{y} = l_a \dot{\theta}
\]

The equation of motion used to find the lift coefficient is given by Equation 11 for which the 3D lift slope is obtained, and Equation 12 is further applied in order to find the 2D sectional lift slope.

\[
\frac{M_m}{l_a} + qS \frac{dC_L}{d\alpha} \alpha = m_w(x - \frac{c}{4})\dot{\beta} + m_w \ddot{y}
\]

\[
\frac{dc_l}{d\alpha} = \frac{\frac{dC_L}{d\alpha}}{1 - \frac{dC_L}{d\alpha} \frac{1}{\pi AR}}
\]

B. Moment Coefficients

The pitching moment \(M_p\) about the quarter-chord may be given as:

\[
M_p = I_w \dot{\beta} - M_x
\]
Figure 9. Velocity history obtained by differentiating reconstructed motion history

Figure 10. Acceleration history obtained by differentiating reconstructed motion history
\( M_s \) is the moment contribution from the servomotor, which may be found by applying Equation 1 for a case at 0 ft/s wind tunnel speed. The aerodynamic pitching moment at 0 m/s wind tunnel speed is assumed to be negligible, so Equation 1 may be used to solve for \( M_s \). The conversion from \( M_p \) into a coefficient is then simply:

\[
c_m = \frac{M_p}{qSc}
\]  

(14)

The moment coefficient is obtained from the experimental case with the main motor driving the arm at 10A and the arm sweep running at 10 ft/s wind speed. The servomotor is disconnected such that the wing pitches freely due only to its rotational inertia and pitching moment.

\[\text{Figure 11. Wing used in the experiment}\]

C. Airfoil Verification

This section describes the method used to verify the NACA number of the airfoil as well as the shape of the wing. Both qualitative and quantitative methods were used to arrive at the conclusion. Figure 11 shows the wing used in the experiment. In the following sections, the tip of the wing is taken to be the top part of the wing.

D. Quantitative Analysis

The wing span was measured to be 18.125 inches. The span was marked into 10 segments. Clear tape was placed at 1.8125 inches intervals from the tip to the root of the wing, measured to 0.001 inch precision with a Vernier Caliper.

Subsequently, the chord length and the thickness were measured at each of these locations as shown in Table 2.

Using the tip of the wing, and using a pen, the shape of the airfoil was traced on a sheet of paper. This shape was compared to the actual NACA 0013 airfoil shape (Airfoil Investigation Database) which is shown in Figure 12.

The red outline is the actual NACA 0013 shape while the black outline was the traced shape. Thus the airfoil shape is seen to be a NACA0013.
Table 2. Table of coordinate measurements

<table>
<thead>
<tr>
<th>Chord No.</th>
<th>Length (inches)</th>
<th>Thickness (inches)</th>
<th>t/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Tip)</td>
<td>5.9</td>
<td>0.764</td>
<td>0.129</td>
</tr>
<tr>
<td>2</td>
<td>5.873</td>
<td>0.759</td>
<td>0.129</td>
</tr>
<tr>
<td>3</td>
<td>5.896</td>
<td>0.764</td>
<td>0.130</td>
</tr>
<tr>
<td>4</td>
<td>5.883</td>
<td>0.76</td>
<td>0.129</td>
</tr>
<tr>
<td>5</td>
<td>5.902</td>
<td>0.752</td>
<td>0.127</td>
</tr>
<tr>
<td>6</td>
<td>5.921</td>
<td>0.753</td>
<td>0.127</td>
</tr>
<tr>
<td>7</td>
<td>5.89</td>
<td>0.757</td>
<td>0.129</td>
</tr>
<tr>
<td>8</td>
<td>5.907</td>
<td>0.764</td>
<td>0.129</td>
</tr>
<tr>
<td>9</td>
<td>5.926</td>
<td>0.785</td>
<td>0.132</td>
</tr>
<tr>
<td>10</td>
<td>5.956</td>
<td>0.808</td>
<td>0.136</td>
</tr>
<tr>
<td>11 (root)</td>
<td>5.927</td>
<td>0.786</td>
<td>0.133</td>
</tr>
<tr>
<td>Average</td>
<td>3.49</td>
<td>0.768</td>
<td>0.130</td>
</tr>
</tbody>
</table>

Figure 12. Comparison between NACA 0013 and the traced airfoil shape
IV. Results

A. Quasi-Steady Lift and Moment Coefficients

The experimentally measured lift coefficient for different angle of attack for the lowest rate of motion of 10 ms motor time delay and at 10 feet per second wind tunnel speed are plotted in Figure 13.

Figure 13. Airfoil Lift Coefficient vs. Effective Angle of attack (-5 to 15) for different Reynolds #

Figure 14 shows the quasi-steady-state lift curve, calculated from low-frequency driving, in the linear and initial stall regimes, compared to results from the XFOIL potential-boundary layer airfoil aerodynamics code, corrected for aspect ratio.

B. Results for High Angle of Attack

Having seen that the data agree reasonably well with the simple prediction method of XFOIL, we now present our data for the high angle of attack regime. The data acquisition and analysis process are exactly the same regardless of angle of attack range, and as mentioned at the outset, our method has the same sensitivity and response over the entire range of angles of attack and rate. Figures 16 and 17 shows the lowest-frequency data for angles of attack of -5° to 15° and 0° to 45° respectively.

Figure 15 shows a comparison between the experimental case, XFOIL simulation and known moment coefficient from the airfoil investigation database. The plot shows the experimental result follows closely to the plot from the database except for angles of attack from 10 to 13 degrees. The reasons for these anomalous results are not determined at this time, but may indicate errors in the inclusion of the inertial pitching moment of the wing.

C. Reynolds Number Effect on Quasi-Steady Results

The data for each of the wind tunnel speeds were taken, and filtered for cases from -5° to 45° angle of attack. The data were then reduced by taking the mean lift coefficient for repeated cases. The lift coefficient of each of these cases takes the aspect ratio of the wing into account.

Two graphs each showing three cases of the airfoil coefficient versus effective angle of attack is for -5° to 15° angle of attack and from 0° to 45° angle of attack is shown in Figure 16 and Figure 17 respectively. The graphs indicate a higher chord Reynolds number results in a relatively overall higher lift coefficient at angles
Figure 14. Static Case

Figure 15. Comparison of Coefficient of Moments
Figure 16. Airfoil Lift Coefficient vs. Effective Angle of attack (-5 to 15) for different Reynolds

Figure 17. Airfoil Lift Coefficient vs. Effective Angle of attack (0 to 45) for different Reynolds
of attacks higher than $15^\circ$. Again, the case at 112,000 Reynolds number is substantially different, with a low value of lift curve slope. The reasons for this are not known at present, but may have to do with transition.

V. Conclusions

In this first paper on the experiment, a double-pendulum wing apparatus is used to extract quasi-steady lift and moment coefficient variations through a substantial range of angles of attack, purely from the dynamics and kinematics of the apparatus rather than from direct force measurements. Specific points shown to-date:

1. Techniques to measure and verify the shape of the test model are described. These are seen to perform with the needed accuracy for such experiments.

2. A method is presented, where transformation to the frequency domain and spectral averaging are used to extract continuous differentiable functions for the motion, in order to extract acceleration data.

3. From measured acceleration histories and inertias, instantaneous forces and moments are obtained.

4. The data at the lowest frequencies of motion are used to construct quasi-steady variations in lift and moment coefficients.

5. The measured quasi-steady lift coefficient variation with angle of attack agrees well with the calculated results from the XFOIL potential-boundary layer airfoil code, corrected for aspect ratio, over the linear range.

6. Through stall and beyond, the experimental measurements give consistent results, though not in agreement with XFOIL in this region where the potential-boundary layer formulation cannot be expected to give good results.

7. Variations of the quasi-steady lift curve with Reynolds number are substantial, in the low Reynolds number regime. The lift curve slope improves with rising Reynolds number.

8. Extraction and validation of the quasi-steady moment coefficient variation are in progress at this writing.

9. Once validated in the quasi-steady regime, the technique appears to have no further impediments to usage in extracting lift and pitching moment variations at much higher rates of change.

10. Thus it appears at this writing that we have developed a viable technique to extract dynamic lift and pitching moment variations over a wide range of angles of attack and reduced frequencies.

Acknowledgments

This work was partly funded under the NASA Extrovert Cross Disciplinary Learning Initiative. Mr. Anthony Springer is the Technical Monitor. Special thanks to Tejas Kotak, Akshay Pendharkar and Julian Forero.
References


VI. Recent Work 2012

Calculated Pitching Moment about the quarter chord of NACA 0013, which is the aerodynamic center of the airfoil. The pitching moment was calculated from the angular acceleration and the inertia of the wing. With the pitching moment known, the moment coefficient can be easily calculated using equation 14.

Cases calculated were experimental runs where the servo motor was disconnected and the wing was allowed to pitch freely due to its rotational inertia and pitching moment. From observation it can be seen that besides the case of 10ms at a wind tunnel speed of 10 feet/second, which is seemingly erratic when compared to the 5ms cases, the 3 results in the 5 ms case indicates a relatively similar repeated pitching moment in the 30 seconds it was allowed to pitch freely. This is shown in Figures 18 to 21.
Figure 18. Moment Coefficient vs. Effective angle of attack (degrees)

Figure 19. Moment Coefficient vs. Effective angle of attack (degrees)
Figure 20. Moment Coefficient vs. Effective angle of attack (degrees)

Figure 21. Moment Coefficient vs. Effective angle of attack (degrees)